Base Tangle Decompositions and Subdivisions of Knots and Links

By M. Ochiai, N.Morimura, and A.Emoto

HOMFLY invariant of Knot or Link

For a knot or link K,

(1) $P(K;x,y) = 1$ *if K is the trivial knot*

(2) $x\,P(K+;x,y) + y\,P(K-;x,y) = P(K\infty;x,y)$

Let T be a 2-string tangle.

$x\,P(T+;x,y) + y\,P(T-;x,y) = P(T\infty;x,y)$

K = T1 + T2, wheren K is a knot and T1 ,T2 are tangles.

$P(T1;\,x,y) = a1\,A1 + b1\,B1$

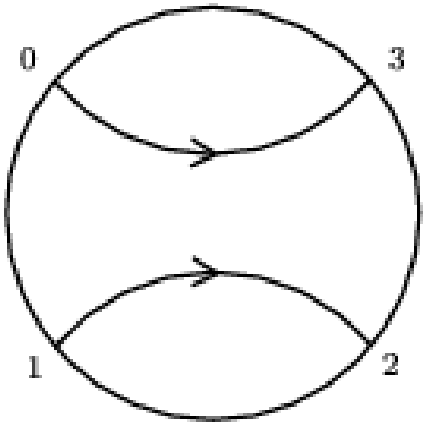
$P(T2;\,x,y) = a2\,A2 + b2\,B2,$

$P(K;x,y)=a1a2(A1+A2)+a1b2(A1+B2)+ b1a2(B1+A2)+b1b2(B1+B2)$

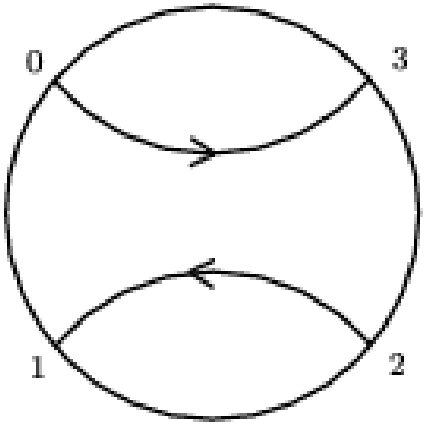
Where A1, B1, A2, B2 are base tangles.

A tangle T is called by a base tangle if

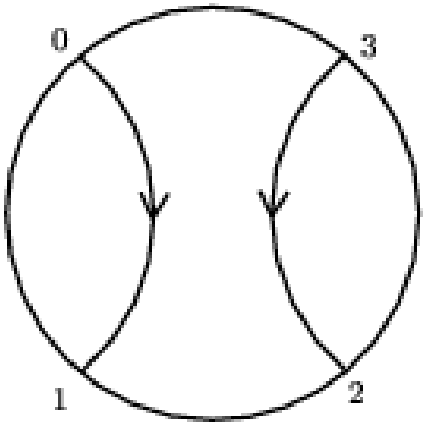
(1) T has minimal crossings with only plus sign.



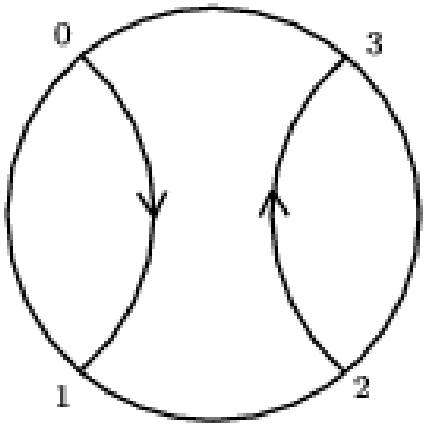
0–型



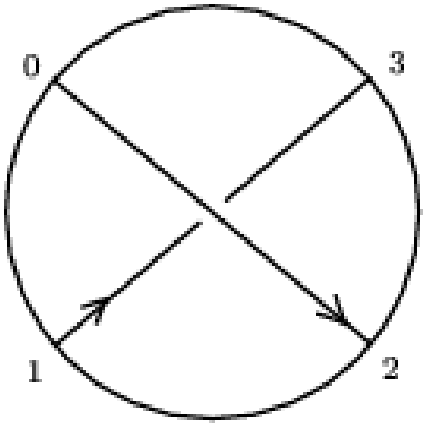
1–型



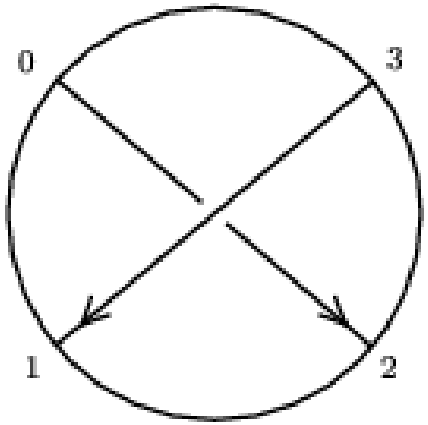
2–型



3–型



4–型



5–型

Tangle decomposition of Conway's knot

$$\left\{\left\{0, 2+\frac{1}{xy^3}-\frac{2}{y^2}-\frac{1}{xy}+\frac{x}{y}\right\}, \left\{4, -y^{-3}+\frac{x}{y^2}\right\}\right\}$$

$$\left\{\left\{4, \frac{1}{x^2y^2}-\frac{1}{xy}-\frac{x}{y}\right\}, \left\{0, \frac{1}{x}-\frac{1}{xy^2}+\frac{2}{y}-\frac{1}{x^2y}\right\}\right\}$$

この出力は以下に示すようなリストである.

$$\{\{A_1, \alpha_1\}, \{B_1, \beta_1\}\}$$

$$\{\{A_2, \alpha_2\}, \{B_2, \beta_2\}\}$$

ただし、$P(T_1, x, y) = \alpha_1 A_1 + \beta_1 B_1$ 、$P(T_2, x, y) = \alpha_2 A_2 + \beta_2 B_2$ である.

$$(x+y)\alpha_1\alpha_2+\alpha_2\beta_1+\alpha_1\beta_2+\left(\frac{1}{x}-\frac{y(x+y)}{x}\right)\beta_1\beta_2$$

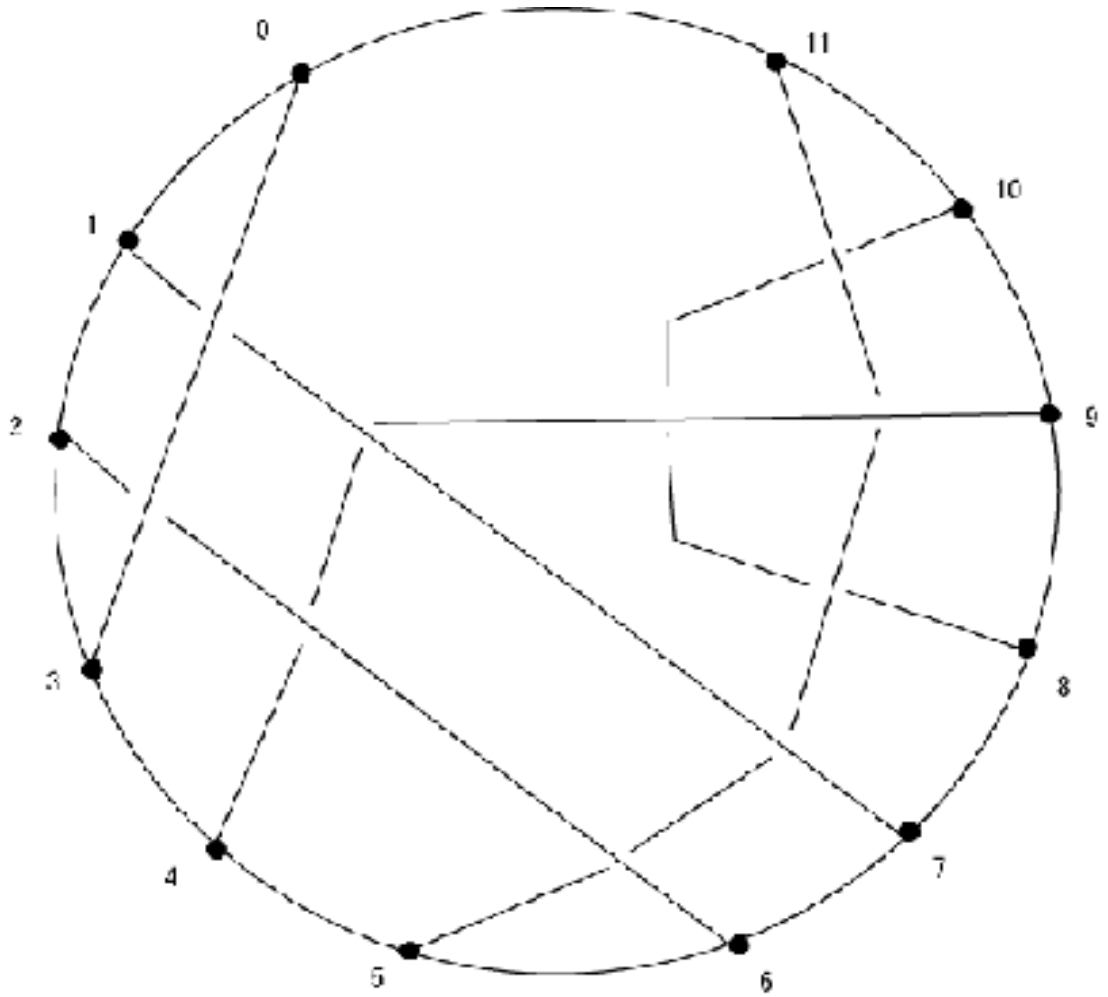
$$= 7-\frac{3}{x^2}+y^{-4}-\frac{1}{x^2y^4}-\frac{1}{x^3y^3}+\frac{6}{xy^3}-\frac{3x}{y^3}-\frac{11}{y^2}+\frac{6}{x^2y^2}+\frac{2x^2}{y^2}+\frac{1}{x^3y}-\frac{11}{xy}+\frac{6x}{y}+\frac{2y}{x}$$

n-tangle decomposition(n>2)

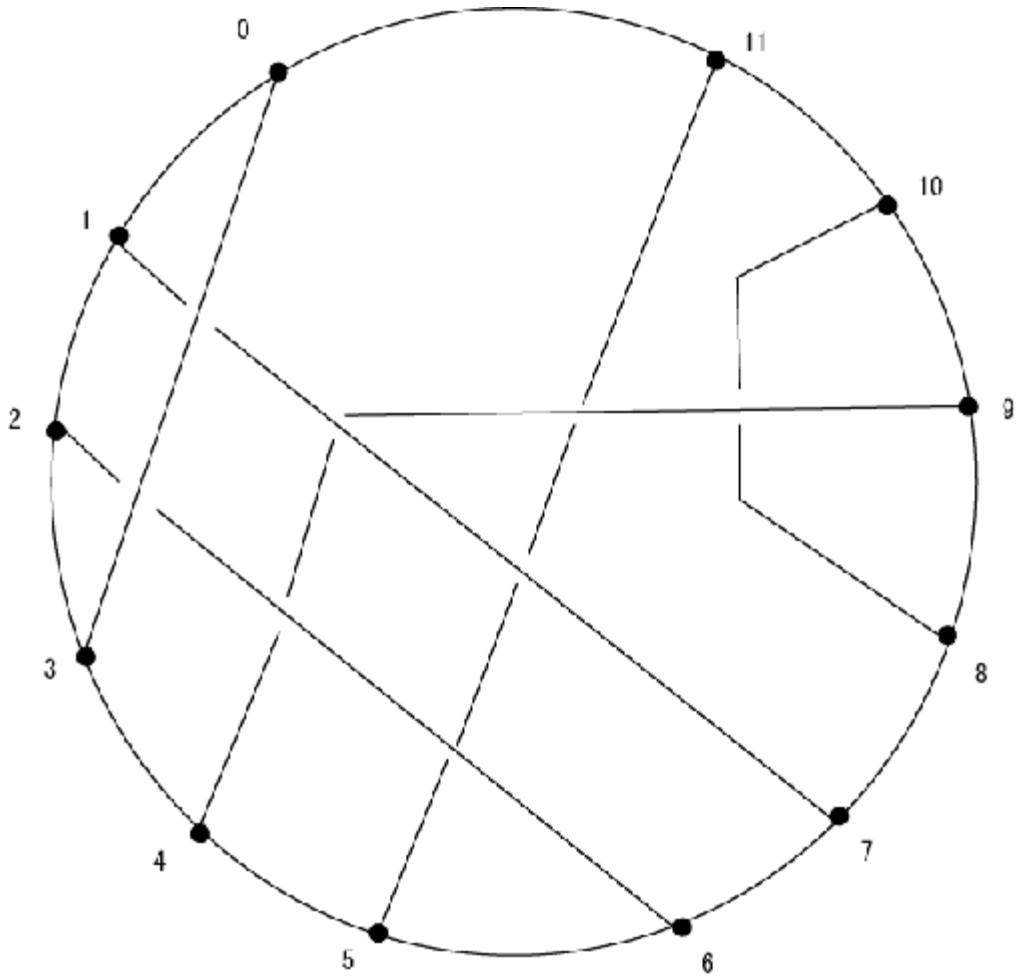
- Base tangle by using **oriented ordered tangle**

(1) Minimal crossings

(2) First string has only over (resp. under) crossings, second string has also over (resp. under) crossings other than crossing with the first string,….



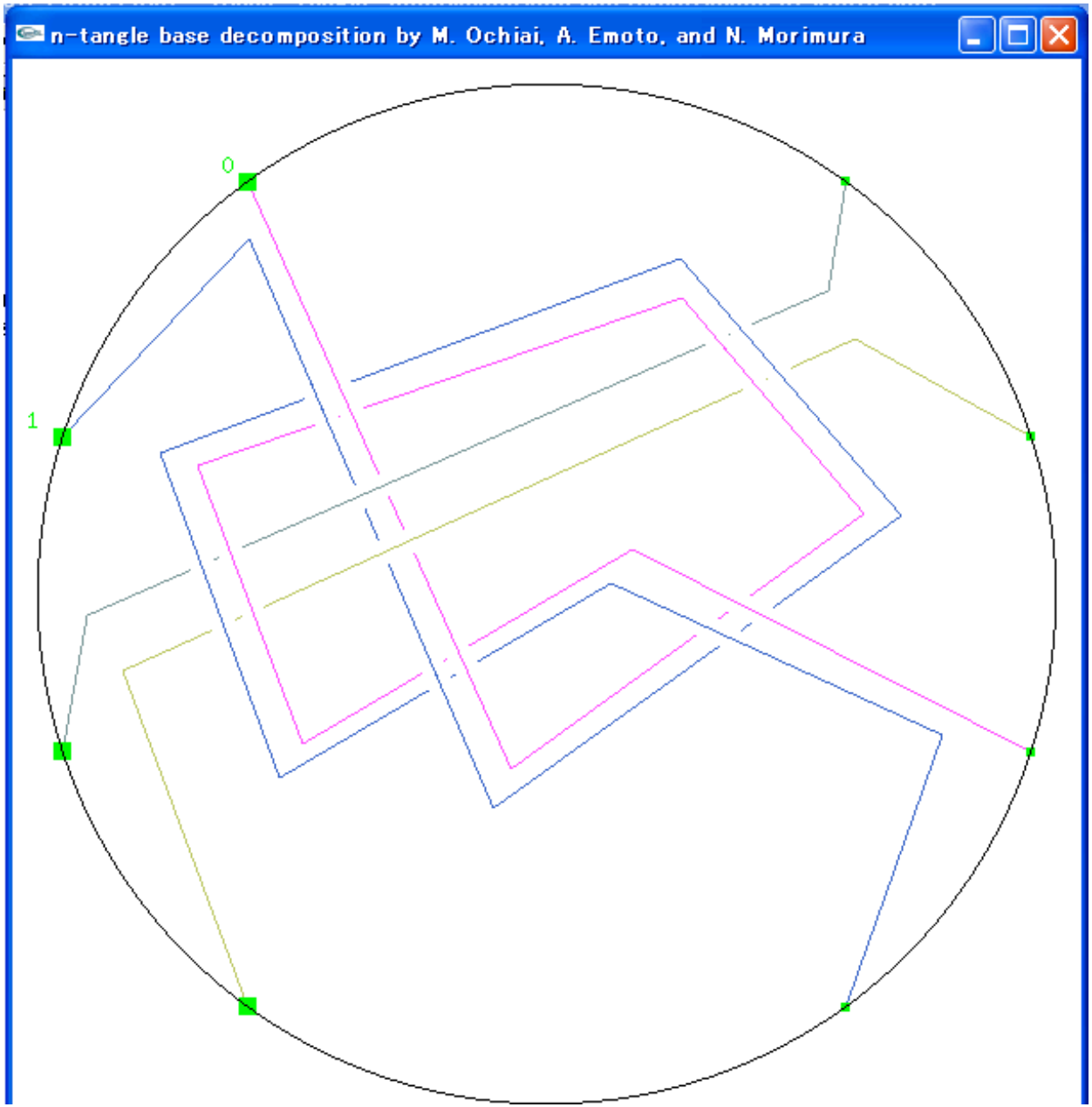
(0, 3, 1, 7, 2, 6, 9, 4, 11, 5, 10, 8) の基底タングルと同値な
非最少交点数を持つタングル

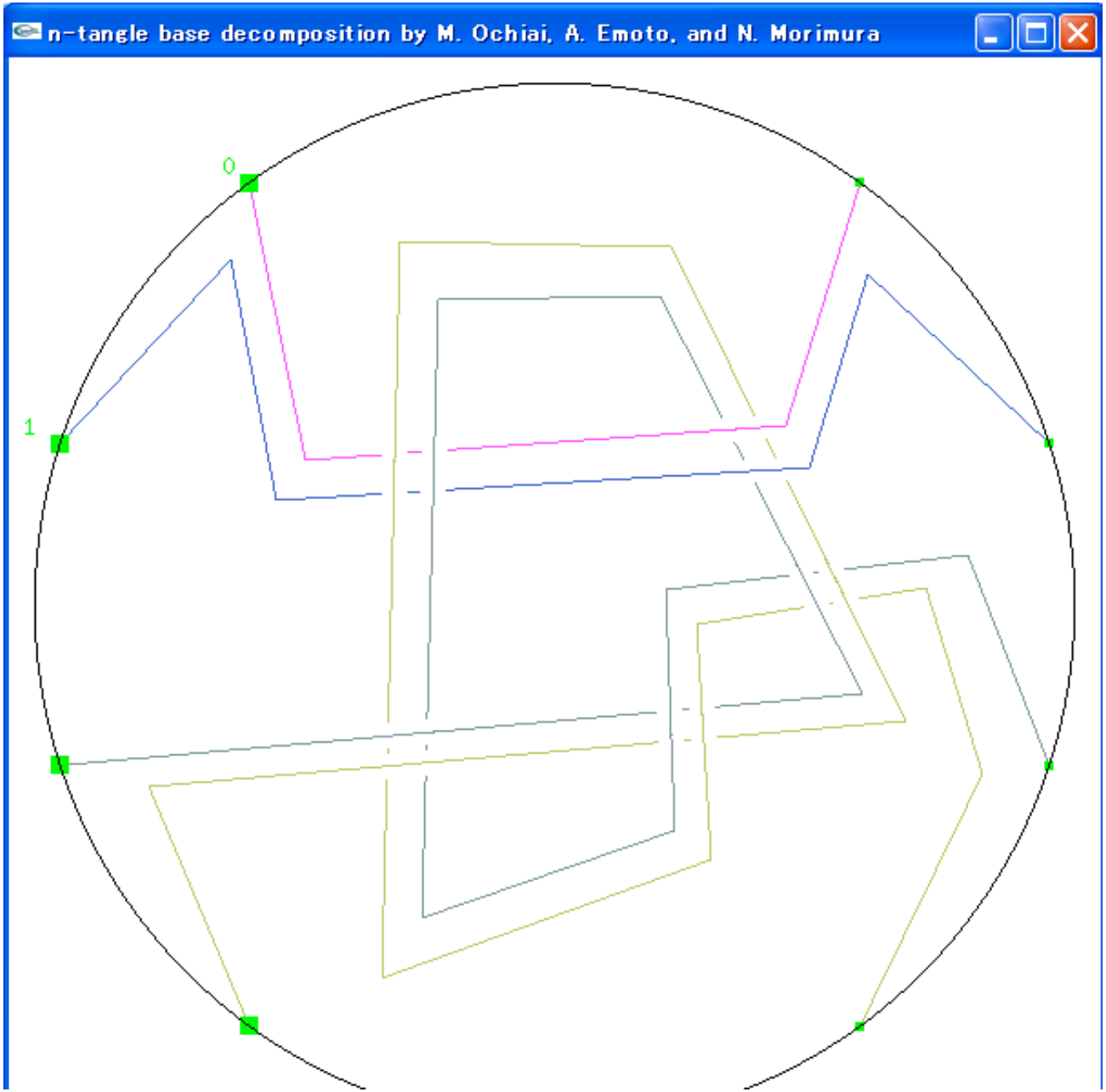


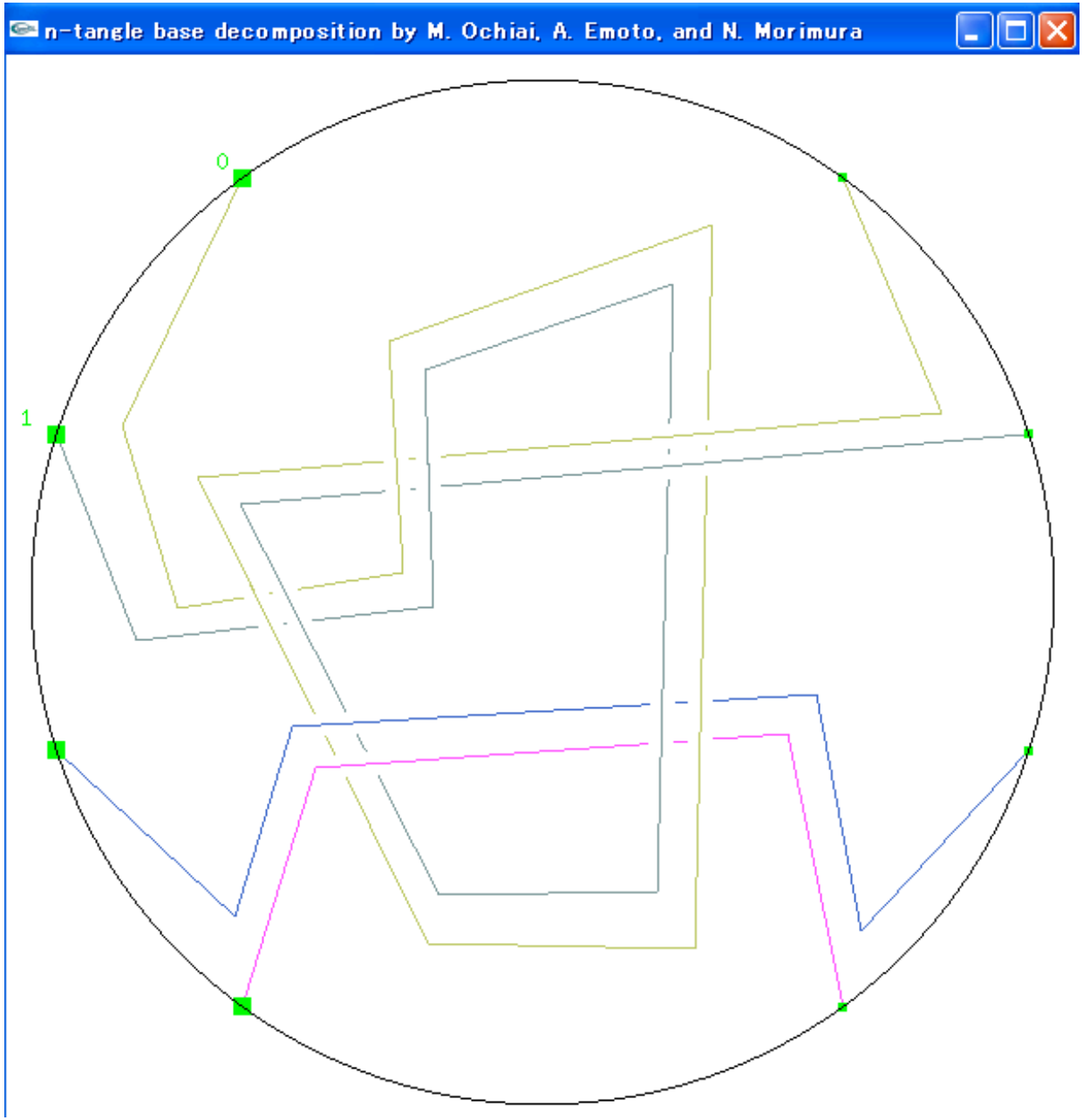
(0, 3, 1, 7, 2, 6, 9, 4, 11, 5, 10, 8)の基底タングル

Reconstruction of a base tangle from a sequence

- Our first method uses to make a base tangle from a sequence by using small deformations of string end points (The last version).
- The second method makes to classify n-multiple points which have models of n double points. This program made by Miss A. Emoto. And this program can make n string base tangles with up to 12. But practically, up to 9-string, in this case there exists 9! base tangles.


n-tangle base decomposition by M. Ochiai, A. Emoto, and N. Morimura
0
1


n-tangle base decomposition by M. Ochiai, A. Emoto, and N. Morimura
0
1


n-tangle base decomposition by M. Ochiai, A. Emoto, and N. Morimura
0
1

BASETANGLE1 - メモ帳

ファイル(F)　編集(E)　書式(O)　表示(V)　ヘルプ(H)

```
 0  8  1  7  2  6  3 11  4 10  5  9
 0  4
 0  8  1  7  2  6  3 11  4 10  5  9
1 6 -6 9 4 -6 -45 3 -7 45 2 -6 60 2 -8 -279 1 -7 -36 1 -9 60 (
 0  8  1  7  2  6  3 11  4  9  5 10
3 4 -7 -15 3 -8 30 2 -7 19 2 -9 -193 1 -8 -8 1 -10 86 0 -7 51
 0  8  1  7  2  6  3 10  4 11  5  9
3 4 -7 -15 3 -8 30 2 -7 19 2 -9 -193 1 -8 -8 1 -10 86 0 -7 51
 0  8  1  7  2  6  3 10  4  9  5 11
3 4 -8 -12 3 -9 45 2 -8 7 2 -10 -218 1 -9 -1 1 -11 188 0 -8 4
 0  8  1  7  2  6  3  9  4 11  5 10
3 4 -8 -12 3 -9 45 2 -8 7 2 -10 -218 1 -9 -1 1 -11 188 0 -8 4
 0  8  1  7  2  6  3  9  4 10  5 11
-3 5 -8 12 4 -9 -30 3 -8 -7 3 -10 148 2 -9 1 2 -11 -86 1 -8 -
 0  8  1  7  2 11  3  6  4 10  5  9
-2 5 -6 -1 4 -5 5 4 -7 -5 3 -6 -4 3 -8 -6 2 -5 48 2 -7 1 2 -9
 0  8  1  7  2 11  3  6  4  9  5 10
-4 3 -7 -2 2 -6 24 2 -8 -1 1 -7 -35 1 -9 -10 0 -6 52 0 -8 18 (
 0  8  1  7  2 11  3 10  4  6  5  9
2 4 -6 1 3 -5 -5 3 -7 5 2 -6 4 2 -8 8 1 -5 -41 1 -7 -1 1 -9 -
 0  8  1  7  2 11  3 10  4  9  5  6
-2 3 -6 -1 2 -5 5 2 -7 -9 1 -6 -4 1 -8 -12 0 -5 58 0 -7 1 0 -
 0  8  1  7  2 11  3  9  4  6  5 10
4 2 -7 4 1 -6 -17 1 -8 1 1 -10 24 0 -9 31 -1 -6 -77 -1 -8 -13
 0  8  1  7  2 11  3  9  4 10  5  6
2 4 -6 1 3 -5 -5 3 -7 5 2 -6 4 2 -8 8 1 -5 -41 1 -7 -1 1 -9 -
```

BASETANGLE2 - メモ帳

ファイル(F)　編集(E)　書式(O)　表示(V)　ヘルプ(H)

```
0 11  1 10  2  9  3  8  4  7  5  6
 0  0
 0 11  1 10  2  9  3  8  4  7  5  6
-1 6 -6 -6 5 -5 18 5 -7 -12 4 -4 96 4 -6 -108 4 -8 -8 3 -3 17
 0 11  1 10  2  9  3  8  4  6  5  7
-4 5 -6 -15 4 -5 70 4 -7 -9 4 -9 72 4 -11 -14 3 -4 238 3 -6 -5
 0 11  1 10  2  9  3  7  4  8  5  6
-4 5 -6 -15 4 -5 70 4 -7 -9 4 -9 24 4 -11 -14 3 -4 238 3 -6 -5
 0 11  1 10  2  9  3  7  4  6  5  8
-6 5 -7 3 5 -9 -38 4 -6 107 4 -8 -24 4 -10 264 4 -12 -47 3 -5
 0 11  1 10  2  9  3  6  4  8  5  7
-6 5 -7 3 5 -9 -38 4 -6 107 4 -8 -24 4 -10 273 4 -12 -47 3 -5
 0 11  1 10  2  9  3  6  4  7  5  8
4 6 -7 15 5 -6 -64 5 -8 6 5 -10 -48 5 -12 14 4 -5 -200 4 -7 42
 0 11  1 10  2  8  3  9  4  7  5  6
-1 6 -7 -4 5 -6 13 5 -8 -4 4 -5 57 4 -7 -71 4 -9 4 4 -11 63 3
 0 11  1 10  2  8  3  9  4  6  5  7
-4 5 -7 -7 4 -6 63 4 -8 -18 4 -10 68 4 -12 130 3 -7 -476 3 -9
 0 11  1 10  2  8  3  7  4  9  5  6
-1 6 -8 -6 5 -7 11 5 -9 -8 4 -6 85 4 -8 -59 4 -10 30 4 -12 136
 0 11  1 10  2  8  3  7  4  6  5  9
-1 6 -9 -14 5 -8 11 5 -10 -6 5 -12 -28 4 -7 152 4 -9 -68 4 -1
 0 11  1 10  2  8  3  6  4  9  5  7
-1 6 -9 -14 5 -8 11 5 -10 -16 5 -12 -28 4 -7 152 4 -9 -78 4 -
 0 11  1 10  2  8  3  6  4  7  5  9
1 7 -9 6 6 -8 -10 6 -10 8 5 -7 -71 5 -9 48 5 -11 -10 5 -13 -10
```

KNOTBYTANGLE - メモ帳

ファイル(F) 編集(E) 書式(O) 表示(V) ヘルプ(H)

```
Base Rule : ( 0)
The first base tangle decomposition:
Base Rule : ( 0)
The second base tangle decomposition:

A HOMFLY polynomial by tangle decomposition:
-4x^6 y^-5 -28x^5 y^-4 +47x^5 y^-6 -88x^4 y^-3 +264x^4 y^-5 -207x^4 y^-7 -156x^3 y^-2 +
+548x^-8 y^-5 -971x^-8 y^-7 +210x^-8 y^-9 -2x^-8 y^-11 -135x^-9 y^-6 +190x^-9 y^-8 -22x

*** A full pdata made by the two tangles. ***
 55  11  38  82  16  60  61  17  83  39   2  46  36  80  49   5   8  52  73  29  32  76
  1   1   2   2   3   3   4   4   1   1   2   2   3   3   4   4   3   3   1   1   2   2
  1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1
*** A pdata made by the two tangles. ***
88 2
44 44
-55 -11 16 60 -83 -39 36 80 8 52 -73 -29 -77 -33 42 86 -65 -21 -47 -3 26 70 -56 -12 15 !

A HOMFLY polynomial by direct computation:
-4x^6 y^-5 -28x^5 y^-4 +47x^5 y^-6 -88x^4 y^-3 +264x^4 y^-5 -207x^4 y^-7 -156x^3 y^-2 +
+548x^-8 y^-5 -971x^-8 y^-7 +210x^-8 y^-9 -2x^-8 y^-11 -135x^-9 y^-6 +190x^-9 y^-8 -22x
```

The base tangle decomposition of the first tangle of Terasaka's knot

BASETANGLE1 - メモ帳

ファイル(F) 編集(E) 書式(O) 表示(V) ヘルプ(H)

```
0  8  1  7  2  6  3 11  4 10  5  9
0  4
0  8  1  7  2  6  3 11  4 10  5  9
1 6 -6  9 4 -6  -45 3 -7  45 2 -6  60 2 -8  -279 1 -7  -36 1 -
0  8  1  7  2  6  3 11  4  9  5 10
3 4 -7  -15 3 -8  30 2 -7  19 2 -9  -193 1 -8  -8 1 -10  86 0
0  8  1  7  2  6  3 10  4 11  5  9
3 4 -7  -15 3 -8  30 2 -7  19 2 -9  -193 1 -8  -8 1 -10  86 0
0  8  1  7  2  6  3 10  4  9  5 11
3 4 -8  -12 3 -9  45 2 -8  7 2 -10  -218 1 -9  -1 1 -11  188 (
0  8  1  7  2  6  3  9  4 11  5 10
3 4 -8  -12 3 -9  45 2 -8  7 2 -10  -218 1 -9  -1 1 -11  188 (
0  8  1  7  2  6  3  9  4 10  5 11
-3 5 -8  12 4 -9  -30 3 -8  -7 3 -10  148 2 -9  1 2 -11  -86 1
0  8  1  7  2 11  3  6  4 10  5  9
-2 5 -6  -1 4 -5  5 4 -7  -5 3 -6  -4 3 -8  -6 2 -5  48 2 -7
0  8  1  7  2 11  3  6  4  9  5 10
-4 3 -7  -2 2 -6  24 2 -8  -1 1 -7  -35 1 -9  -10 0 -6  52 0 -
0  8  1  7  2 11  3 10  4  6  5  9
2 4 -6  1 3 -5  -5 3 -7  5 2 -6  4 2 -8  8 1 -5  -41 1 -7  -1
0  8  1  7  2 11  3 10  4  9  5  6
-2 3 -6  -1 2 -5  5 2 -7  -9 1 -6  -4 1 -8  -12 0 -5  58 0 -7
0  8  1  7  2 11  3  9  4  6  5 10
```

The base tangle decomposition of the second tangle of Terasaka's knot

BASETANGLE2 - メモ帳

ファイル(F)　編集(E)　書式(O)　表示(V)　ヘルプ(H)

```
0 11  1 10  2  9  3  8  4  7  5  6
0  0
0 11  1 10  2  9  3  8  4  7  5  6
-1 6 -6  -6 5 -5  18 5 -7  -12 4 -4  96 4 -6  -108 4 -8
0 11  1 10  2  9  3  8  4  6  5  7
-4 5 -6  -15 4 -5  70 4 -7  -9 4 -9  72 4 -11  -14 3 -4
0 11  1 10  2  9  3  7  4  8  5  6
-4 5 -6  -15 4 -5  70 4 -7  -9 4 -9  24 4 -11  -14 3 -4
0 11  1 10  2  9  3  7  4  6  5  8
-6 5 -7  3 5 -9  -38 4 -6  107 4 -8  -24 4 -10  264 4 -12
0 11  1 10  2  9  3  6  4  8  5  7
-6 5 -7  3 5 -9  -38 4 -6  107 4 -8  -24 4 -10  273 4 -12
0 11  1 10  2  9  3  6  4  7  5  8
4 6 -7  15 5 -6  -64 5 -8  6 5 -10  -48 5 -12  14 4 -5  -
0 11  1 10  2  8  3  9  4  7  5  6
-1 6 -7  -4 5 -6  13 5 -8  -4 4 -5  57 4 -7  -71 4 -9  4
0 11  1 10  2  8  3  9  4  6  5  7
-4 5 -7  -7 4 -6  63 4 -8  -18 4 -10  68 4 -12  130 3 -7
0 11  1 10  2  8  3  7  4  9  5  6
-1 6 -8  -6 5 -7  11 5 -9  -8 4 -6  85 4 -8  -59 4 -10  3
0 11  1 10  2  8  3  7  4  6  5  9
-1 6 -9  -14 5 -8  11 5 -10  -6 5 -12  -28 4 -7  152 4 -9
0 11  1 10  2  8  3  6  4  9  5  7
```

The 3-parallel HOMFLY polynomial of Terasaka's knot

KNOTBYTANGLE - メモ帳

ファイル(F) 編集(E) 書式(O) 表示(V) ヘルプ(H)

```
*** A full pdata of a 3-parallel tangle made by a 2-tangle. ***
 70  43  16  87  96 105  24  51  78  79  52  25 106  97  88   3  30  57  84  93 102  61  34
  1   1   1   2   2   2   3   3   3   4   4   4   1   1   1   2   2   2   3   3   3   4   4
  1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1
Base Rule : ( 0)
The first base tangle decomposition:
( 2414.547000 [sec] )
*** A full pdata of a 3-parallel tangle made by a 2-tangle. ***
 79  55  31  30  54  78  80  56  32  29  53  77  81  57  33  28  52  76  82  58  34  39  63
  4   4   4   3   3   3   4   4   4   3   3   3   4   4   4   3   3   3   1   1   1   2   2
  1   1   1   1   1   1   2   2   2   2   2   2   3   3   3   3   3   3   4   4   4   4   4
Base Rule : ( 0)
The second base tangle decomposition:
( 2261.641000 [sec] )

A 3-parallel HOMFLY polynomial by tangle decomposition:
-8x^11y^-9-88x^10y^-8+246x^10y^-10+x^10y^-12-452x^9y^-7+2292x^9y^-9-3869x^9y^-11-122x^9y^-13-
18x^1y^-1-405131x^1y^-3+4112972x^1y^-5-15197174x^1y^-7+25673235x^1y^-9-48099290x^1y^-11+11925
1-48038573x^-3y^-23+8027437x^-3y^-25+9916x^-3y^-27-478276x^-4y^-2+19258906x^-4y^-4-192150319x
23+65761145x^-7y^-25-15183756x^-7y^-27+645023x^-7y^-29+9x^-7y^-31+377x^-8y^-4-134098085x^-8y^
128x^-11y^-25-869806x^-11y^-27+1585541x^-11y^-29-96814x^-11y^-31+565x^-11y^-33+9517x^-12y^-8-
15y^-33-2x^-15y^-35-2715x^-16y^-12-46982031x^-16y^-14-5487657x^-16y^-16+328177727x^-16y^-18-3
22y^-30+45x^-23y^-21+162x^-23y^-23-439x^-23y^-25+230x^-23y^-27-x^-24y^-22-4x^-24y^-24+10x^-24
```

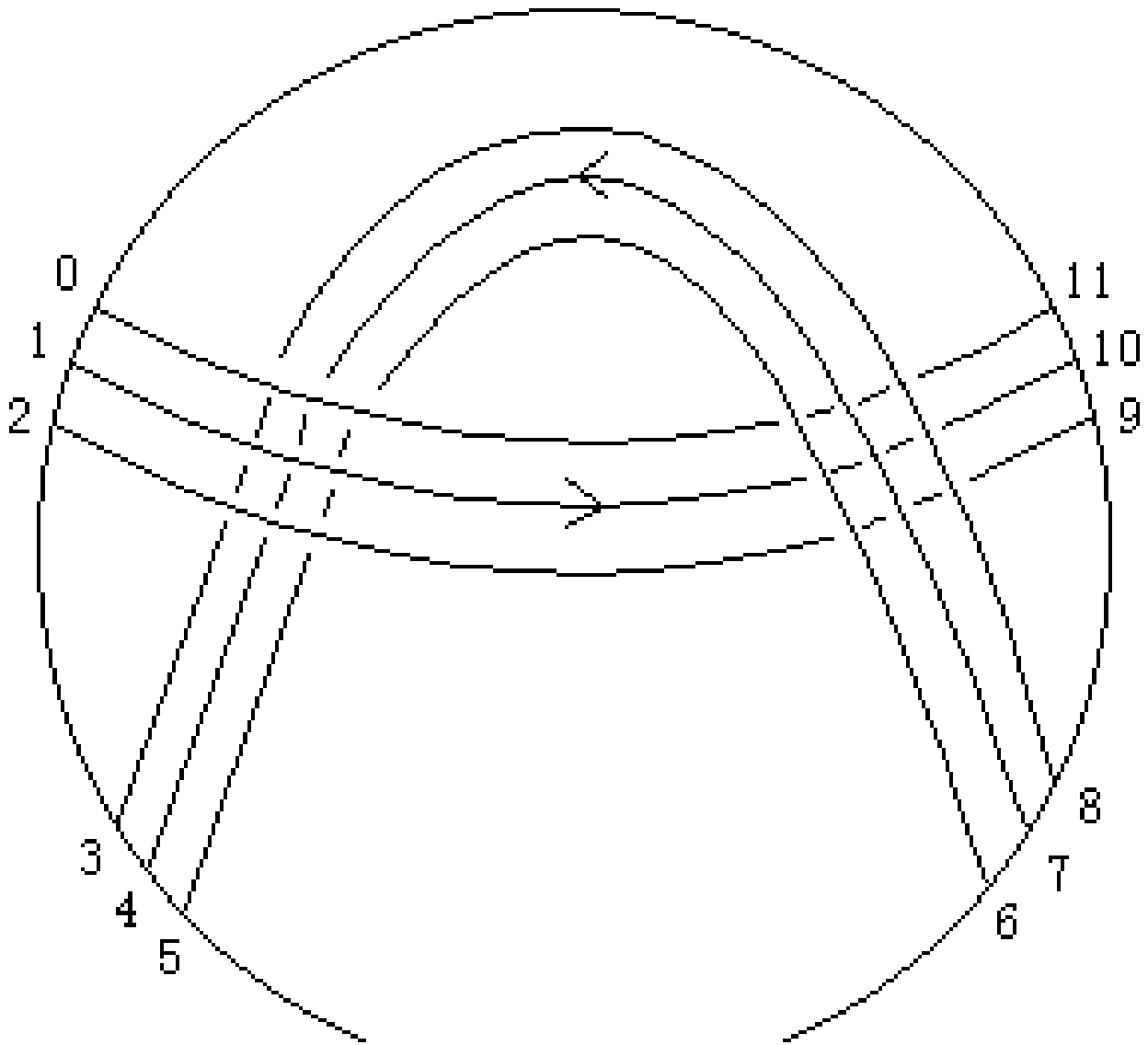
Final computation of base tangle decomposition

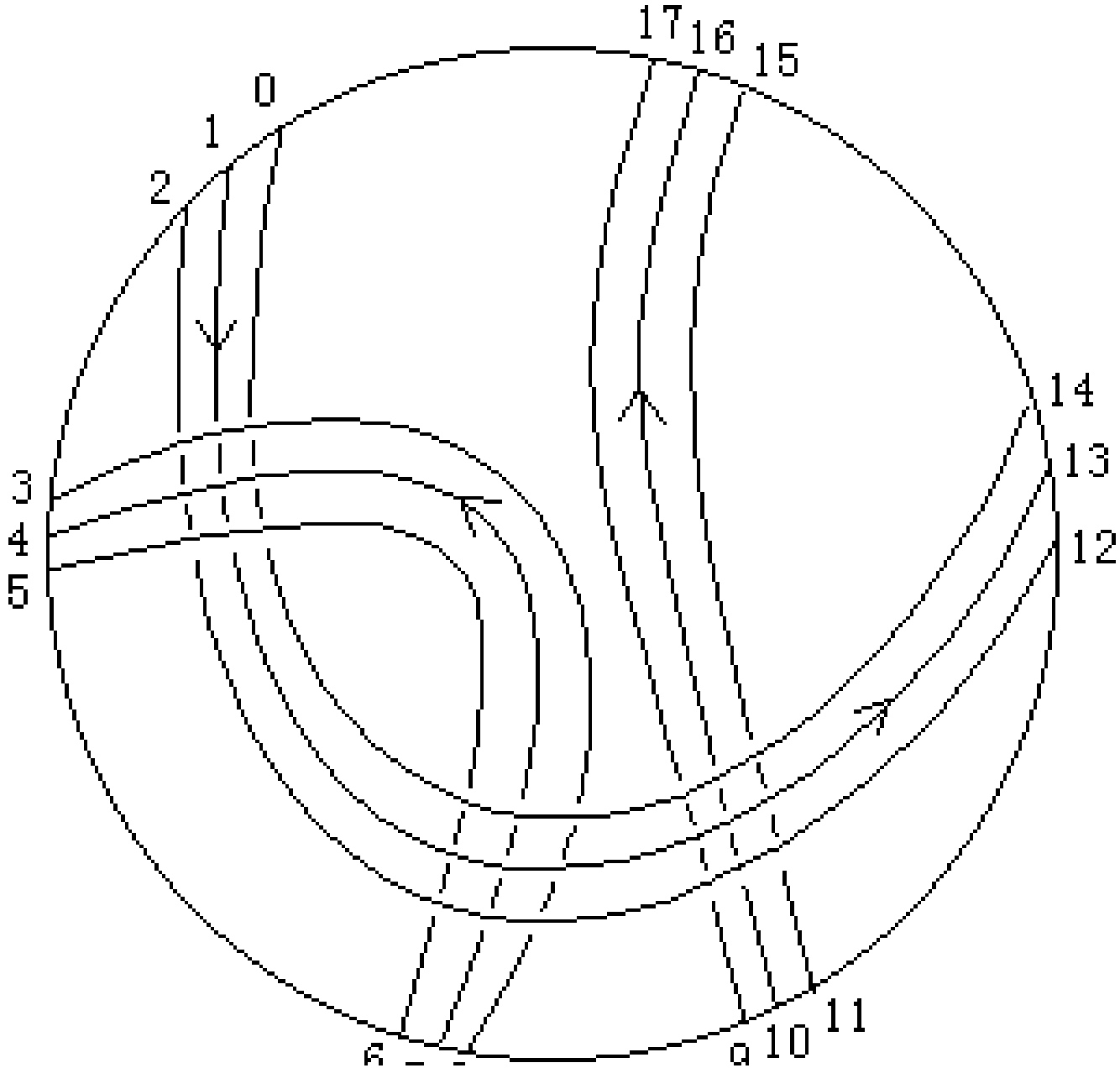
"e:¥NETTangle64¥Debug¥Tangle64.exe"

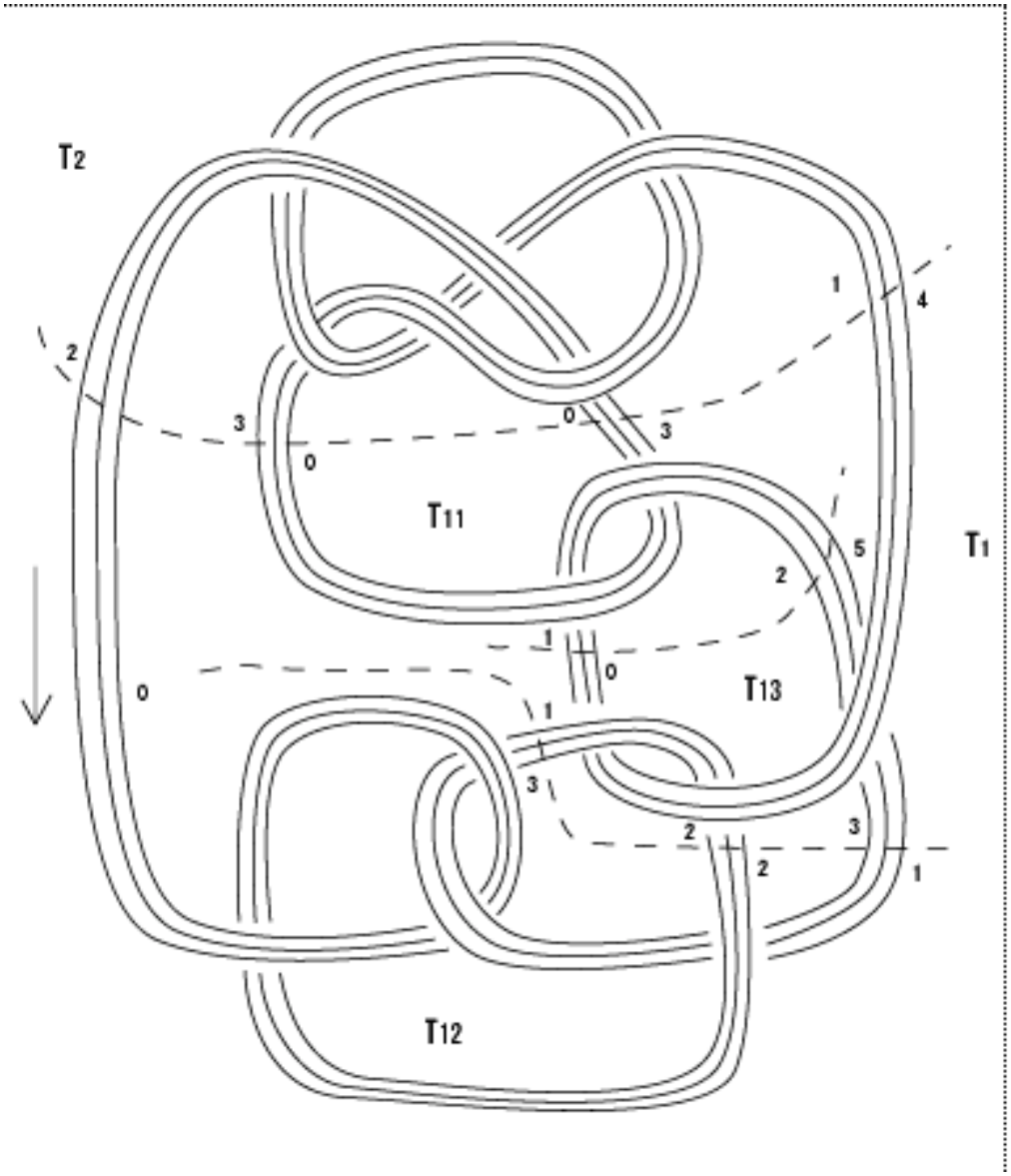
```
708-th base tangle has matched.
709-th base tangle has matched.
710-th base tangle has matched.
711-th base tangle has matched.
712-th base tangle has matched.
713-th base tangle has matched.
714-th base tangle has matched.
715-th base tangle has matched.
716-th base tangle has matched.
717-th base tangle has matched.
718-th base tangle has matched.
719-th base tangle has matched.

A 3-parallel HOMFLY polynomial by tangle decomposition:
-8x^11y^-9-88x^10y^-8+246x^10y^-10+x^10y^-12-452x^9y^-7+2292x^9y^-9-3869x^9y^-11
-122x^9y^-13-5x^9y^-15-1416x^8y^-6+10179x^8y^-8-29683x^8y^-10+36290x^8y^-12+1901
x^8y^-14+53x^8y^-16-2982x^7y^-5+28533x^7y^-7-108943x^7y^-9+242961x^7y^-11-225335
x^7y^-13-13701x^7y^-15+278x^7y^-17-4398x^6y^-4+56148x^6y^-6-260874x^6y^-8+748689
x^6y^-10-1382462x^6y^-12+994195x^6y^-14+71452x^6y^-16-1479x^6y^-18-4597x^5y^-3+8
1066x^5y^-5-466519x^5y^-7+1493434x^5y^-9-3700672x^5y^-11+5752416x^5y^-13-3200561
x^5y^-15-316092x^5y^-17+416x^5y^-19-3374x^4y^-2+86565x^4y^-4-662591x^4y^-6+23357
20x^4y^-8-6141136x^4y^-10+13861587x^4y^-12-17928260x^4y^-14+7384816x^4y^-16+1147
919x^4y^-18+9536x^4y^-20-1687x^3y^-1+67170x^3y^-3-750111x^3y^-5+3240874x^3y^-7-8
066767x^3y^-9+19655564x^3y^-11-40318096x^3y^-13+41991733x^3y^-15-11420564x^3y^-1
7-3066180x^3y^-19-19554x^3y^-21-542x^2+36417x^2y^-2-650371x^2y^-4+4033701x^2y^-6
```

How to compute the 3-parallel HOMFLY polynomial of the following knot with 15-crossings


0
1
2
3
4
5
6
7
8
9
10
11


17
16
15
0
1
2
14
13
3
4
12
5
11
10
9


T2
T11
T1
T13
T12
1
4
2
3
0
3
0
5
2
1
0
0
1
3
2
3
2
1

Obstruction of subdivision

- Usual base decompositions are applied to 2 different strings and so no “free loops” are generated.
- Tangle subdivision generates at most three free loops.

Subdivision programs

composition(0, 12, 12, 12, 0, 0, NULL);

>> knot , link

composition(1, 12, 18, 9, p, q, base);

composition 696() >> 9-string

composition(2, 12, 18, 6, p, q, base);

composition699() >> 6-string

composition(3, 12, 12, 6, p, q, base);

composition666() >> 6-string

composition(4, 18, 18, 6, p, q, base);

composition9912() >> 6-string