An Application of a Temporal Linear Logic to Timed Petri Nets

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Abstract. It is well known that the Petri net *reachability* is equivalent to the *provability* for the corresponding sequent of linear logic. It is a system which has expressive powers of *resources*, but does not provide a concept of *time* to encode *timed Petri nets* naturally. So we introduce a resource-conscious and time-dependent system, that is, *temporal linear logic*. The aim of the paper is to show the (discrete) timed Petri net reachability is equivalent to the provability in the subsystem of temporal linear logic for the corresponding sequent. Our final target is to analyze the dynamic behavior of timed Petri nets by means of the logic.

1 Introduction

Linear logic which was introduced by Girard in 1987 [3] has been called a resource conscious logic. The expressive power is evidenced by some very natural encodings of computational models such as Petri nets (PN) [7, 15, 8, 10]. Timed Petri nets (TPN) [2] are place-transition nets enhanced with a definite notion of time. In this paper, we focus only on discrete time and describe the relationship between timed Petri nets and logic. Although TPN can be simulated by PN [12], linear logic does not provide natural encodings of TPN since it lacks a concept of time. The aim of the paper is to show the equivalence between (discrete) timed Petri net reachability and the provability for the corresponding sequent in (the subsystem of) temporal linear logic, which is obtained by introducing "time expression" to linear logic.

In [4], although an extension of linear logic with certain features of temporal logic [11] was introduced, it is not enough to represent TPN since it lacks operators to express the difference between the reusable resources such as transitions and the anytime-but-once usable resources (which will disappear after they are used) such as available tokens in TPN. In [14], TPN was considered as a timed extension to quantales [1] which are called *timed R-monoids*, and the logic system, which has TPN as its models, was introduced. TPN is a sound model of the logic, which means that if a sequent of the logic system is provable then the corresponding marking is reachable from the corresponding initial marking of TPN. But the converse does not hold, that is, it is not a complete model of the logic. This is because of using non logical axioms in the deduction, which indicate transitions of TPN.

Our "temporal linear logic" provides a natural encoding of TPN since it has sufficient modalities to represent resources and time concepts of TPN, and also satisfies completeness theorem (Theorem 11). This theorem means that the reachability of TPN is equivalent to the provability of the corresponding sequent. We obtain the decidability of the reachability problem for TPN by means of another method different from [12] replacing TPN into PN.

2 Petri Nets and Linear Logic

Let (Pl, Tr, Ar) be a Petri net where Pl is a finite set of places, Tr is a finite set of transitions and Ar indicates weight of arcs. It is known that the reachability problem for PN is equivalent to the provability problem for the !-Horn fragment of linear logic; [7]

Theorem 1 (Kanovich [7]). *For a given Petri net (Pl, Tr, Ar), a marking M is reachable from a marking M_0 if and only if the following !-Horn sequent*

$$M_0^*, !Tr^* \rightarrow M^*$$

is provable in Linear Logic, where M_0^*, M^* are corresponding formulas and Tr^* is a corresponding sequence of formulas. ∎

Remark. The Horn fragment of linear logic is NP-complete. [5, 6]

3 Timed Petri Nets and Temporal Linear Logic

3.1 Timed Petri Nets

We consider place timed Petri nets in the paper.

Definition 2 (Timed Petri Net). A *(place) Timed Petri Net (TPN)* is a tuple (Pl, Tr, Ar, θ), where:

Pl: Finite set of places
Tr: Finite set of transitions (disjoint with Pl)
Ar: $(Pl \times Tr) \cup (Tr \times Pl) \longrightarrow \mathbf{N}$ (Weight of arcs)
θ: $Pl \longrightarrow \mathbf{N}$

Here $\mathbf{N}$ means the set of natural numbers (including 0). $\theta(p) \geq 0$ indicates the waiting time of the pending token to be an active token in $p \in Pl$. ∎

A multiset of places (i.e. marking) is not sufficient to represent a state of TPN. We need not only the informations of available tokens (i.e. *active tokens*), but also tokens to be usable in future (i.e. *pending tokens*).

Definition 3 (State). A *state* of TPN is a infinite sequence of multisets of places $\langle M_0, M_1, M_2, \ldots \rangle$ where $M_m = M_{m+1} = \ldots = \emptyset$ for some $m \geq 0$. ∎

In a state S at some instant, M_0, which is called a *timed marking*, indicates active tokens and M_i $(i \geq 1)$ indicates pending tokens which will be active after i passage of time.

We will define reachability with respect to states. A reached state is derived by *firing derivation* or *time derivation*.

Definition 4 (Derivation). Let $S = \langle M_0, M_1, M_2, \ldots \rangle$ be a state at some instant t.

firing derivation : We say that a transition τ *is enabled* at S if and only if $M_0^- \subseteq M_0$. Here, M_0^- is a multiset of *input places* to τ. If a transition τ is enabled and we fire it at that instant, the reached state at the same instant t is the state S' defined by

$$S' = \langle M_0 - M_0^- \uplus M_0^+, M_1 \uplus M_1^+, M_2 \uplus M_2^+, \ldots \rangle.$$

Here, M_i^+ indicates a multiset of *output places* p's from τ with $\theta(p) = i \geq 0$. $\uplus$ indicates a multiset union. Note that a firing terminates at the instant. The described derivation is denoted by the notation $S[\tau\rangle S'$.

time derivation : The reached state at the instant $t+1$ from S is the state S' defined by

$$S' = \langle M_0 \uplus M_1, M_2, \ldots \rangle.$$

The described derivation is denoted by the notation $S[\delta\rangle S'$. ∎

We consider TPN in Fig.1. The numbers beside each place p_i indicates $\theta(p_i)$. The state $S = \langle \{p_1, p_2, p_2\}, \emptyset, \ldots \rangle$. The transition τ_1 is enabled at S. We fire it at $t = 0$, the reached state $S_1 = \langle \{p_1\}, \emptyset, \{p_3, p_3\}, \emptyset, \ldots \rangle$ at $t = 0$. After 2 passage of time, the reached state $S_2 = \langle \{p_1, p_3, p_3\}, \emptyset, \ldots \rangle$ at $t = 2$. The transition τ_2 is enabled at S_2 and we fire it at $t = 2$, the reached state $S' = \langle \{p_1, p_3\}, \{p_2\}, \emptyset, \ldots \rangle$ at $t = 2$.

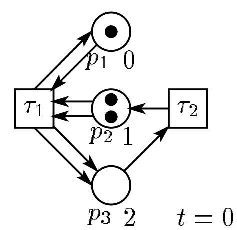

Fig.1: Timed Petri net

Now, we define the reachability for TPN with respect to states. For a derivation sequence $\sigma = \kappa_1 \ldots \kappa_n$ $(n \geq 0)$, we use the notation $S[\sigma\rangle S'$ instead of $S[\kappa_1\rangle S_1[\kappa_2\rangle S_2 \ldots S_{n-1}[\kappa_n\rangle S'$, where κ_i is either $\tau \in Tr$ or δ. Specially, if the number of δ in σ is t, we use the notation $S \overset{t}{[\sigma\rangle} S'$, which means that S' will be reached from S *after t passage of time*. For example, $S \overset{2}{[\tau_1 \delta \delta \tau_2\rangle} S'$ for the TPN in Fig.1.

Definition 5 (Reachable). Let S and S' be states of a TPN. We say that S' is *reachable* from S, which will be denoted by $S' \in [S\rangle$, iff there exists a derivation sequence σ such that $S[\sigma\rangle S'$. ∎

Specially, we say that S' is *strictly reachable* from S at the instant t, which will be denoted by $S' \in [S\rangle_t$, iff there exists a derivation sequence σ such that $S \overset{t}{[\sigma\rangle} S'$.

3.2 Temporal Linear Logic

Now, we introduce temporal linear logic which has several kinds of modalities. Let us A means some resource. Modalities in temporal linear logic express;

$\Box A$ (anytime A): "A can be used at any time but only once. After use, it disappears ". Therefore, an active token in $p \in Pl$ can be represented by $\Box p$.

$\bigcirc A$ (next A): "A can be used only at the next time. After use, it disappears". Therefore, a pending token in $p \in Pl$, which will be active after n passage of time can be represented by $\bigcirc^n \Box p$, where $\bigcirc^n \Box p$ indicates $\underbrace{\bigcirc \ldots \bigcirc}_{n \text{times}} \Box p$.

$!A$ (reusable A): "A can be used always. Never disappears". That is, A means a reusable resource like in linear logic. This is used for a transition.

The expressive power of temporal linear logic is helpful to express the states, transitions and passage of time of timed Petri nets. We will explain it in the following section.

Definition 6 (ITLL). We define a sequent calculus of propositional *intuitionistic temporal linear logic* **ITLL** as a system in Appendix A. ∎

Remark. Temporal linear logic includes linear logic as a subsystem.

ITLL satisfies the cut elimination theorem, that is, if a sequent $\Gamma \rightarrow C$ is provable in **ITLL** then it is provable in **ITLL** without (*cut*) rule. The cut elimination theorem concludes the subformula property.

ITLL is sound and complete for a temporal phase structure model (which is similar to the model in [4]).

Let A, B be formulas. The followings are several syntactical remarks on **ITLL**.

- $\mathbf{1} \otimes A = A \otimes \mathbf{1} = A$.
- $!!A =!A$.
- $!A \rightarrow \Box A$ is provable, but $\Box A \rightarrow !A$ is not provable.
- $\Box A \rightarrow \bigcirc^n A$ is provable, but $\bigcirc^n A \rightarrow \Box A$ is not provable ($n \geq 0$).
- Neither $\bigcirc A \rightarrow A$ nor $A \rightarrow \bigcirc A$ are provable.

"$A = B$" in the list means both $A \rightarrow B$ and $B \rightarrow A$ are provable in **ITLL**.

4 Reachability and Provability

We can encode the reachability problem for TPN into the provability problem of the corresponding Horn sequent of a *Horn-like* system **HTPN** completely. **HTPN** is extended without destroying the equivalence to the reachability problem for TPN in order to associate with temporal linear logic. At the end of this section, we obtain Theorem 11, which claims that the reachability problem for TPN is equivalent to the provability problem for the Horn fragment of the subsystem of temporal linear logic.

At first, we define **HTLL** which include **HTPN** as a subsystem of it. We start from a constructive definition. For atomics $p, q, \ldots$, a *token formula* and a *simple product* are defined by:

$$\alpha ::= \Box p \mid \circ\alpha, \qquad M ::= \alpha \mid M \otimes M,$$

respectively. A token in $p \in Pl$ can be represented by a token formula, a state can be represented by a simple product. Let us consider the encoding for TPN in Fig.1 (See subsection 3.1). For a state S, we denote the corresponding simple product by S^*. In Fig.1, $S^* = \Box p_1 \otimes \Box p_2 \otimes \Box p_2$. The encoding of a transition τ is denoted by τ^*. $\tau_1^* = \Box p_1 \otimes \Box p_2 \otimes \Box p_2 \multimap \Box p_1 \otimes \circ^2 \Box p_3 \otimes \circ^2 \Box p_3$, $\tau_2^* = \Box p_3 \multimap \circ \Box p_2$. In this paper, simple products are denoted by $X, Y, Z, M, \ldots$. For $t \geq 0$, a *Horn sequent* is a sequent of the form

$$\Gamma; \Delta, \mathbf{1} \otimes M \to \circ^t Z,$$

where Γ is a *set* of formulas of the form $X \multimap Y$ and Δ is a *multiset* of formulas of the form $X \multimap Y$. M will associate with the initial state, Z the goal state, Γ the whole transitions in TPN and Δ the used transitions for the derivation sequence.

By a Horn sequent, we can express the statement with respect to the reachability. For the TPN in Fig.1, the statement "S' is reachable from S after 2 passage of time" is represented by the following Horn sequent;

$$\tau_1^*, \tau_2^*; \mathbf{1} \otimes \Box p_1 \otimes \Box p_2 \otimes \Box p_2 \to \circ^2(\mathbf{1} \otimes \Box p_1 \otimes \Box p_3 \otimes \circ \Box p_2). \tag{1}$$

$\mathbf{1}$ in a Horn sequent is a trick to be able to construct the corresponding state from a formula of the form $\circ^n \Box p$. For example, although we can construct the state $\langle \{p_2\}, \emptyset, \ldots \rangle$ from $\circ^3(\mathbf{1} \otimes \Box p_2)$, we cannot decide the corresponding state from the form $\circ^n \Box p$ on the right side of the Horn sequent.

Now, we define **HTLL** as follows;

Definition 7 (HTLL). Let formulas be of the form $X, X \multimap Y, \circ^n Z$, sequents be the form of Horn sequents. We define **HTLL** as a system constructed from Table 1. ∎

We call the subsystem which is constructed by (Ax_1), $(fire)$ and $(next)$ only as **HTPN**. It is not difficult to show the following lemma.

Lemma 8. *Let (Pl, Tr, Ar, θ) be a timed Petri net and S, S' states of it. Then $S \overset{t}{[\sigma\rangle} S'$ for some derivation sequence σ if and only if the following sequent*

$$Tr^*; \mathbf{1} \otimes S^* \to \circ^t(\mathbf{1} \otimes S'^*)$$

is provable in **HTPN**, *where Tr^* is a sequence of τ^* such that $\tau \in Tr$.* ∎

$$\frac{\Gamma; \Delta, Y \otimes M \to \bigcirc^t Z}{\Gamma; \Delta, X \otimes M \to \bigcirc^t Z}\ (fire)$$
provided that $X \multimap Y \in \Gamma$.

$$\frac{}{\Gamma; X \to X}\ (Ax_1)$$

$$\frac{\Gamma; \mathbf{1} \otimes M^{\Box} \otimes \alpha_1 \otimes \ldots \otimes \alpha_k \to \bigcirc^t Z}{\Gamma; \mathbf{1} \otimes M^{\Box} \otimes \bigcirc\alpha_1 \otimes \ldots \otimes \bigcirc\alpha_k \to \bigcirc^{t+1} Z}\ (next)$$
where $M^{\Box}$ is of the form $\Box p_1 \otimes \ldots \otimes \Box p_m$, each α_i is a token formula.

$$\frac{}{\Gamma; X \multimap Y, X \to Y}\ (Ax_2) \qquad \frac{}{\Gamma; \to \mathbf{1}}\ (1) \qquad \frac{\Gamma; \Delta, X \to X}{\Gamma; \Delta, X \otimes Y \to X \otimes Y}\ (\otimes)$$

$$\frac{\Gamma; A, \Delta, M \to \bigcirc^t Z}{\Gamma; \Delta, M \to \bigcirc^t Z}\ (absorb)$$
provided that $A \in \Gamma$.

$$\frac{\Gamma'; \Delta_1, M \to X \quad \Gamma; \Delta_2, X \to \bigcirc^t Z}{\Gamma'; \Delta_1, \Delta_2, M \to \bigcirc^t Z}\ (Hcut)$$
where $\Gamma \subseteq \Gamma'$.

Table 1. Horn temporal linear logic

Lemma 8 claims that we can encode the reachability problem for TPN into the provability problem of the corresponding Horn sequent of **HTPN** completely. For example, since $S\ [\tau_1 \delta\delta \tau_2\overset{2}{\rangle}\ S'$ in Fig.1, the Horn sequent (1) is provable in **HTPN** by Lemma 8. In fact, the following is the proof figure;

$$\cfrac{\cfrac{\cfrac{\cfrac{\tau_1^*, \tau_2^*; \mathbf{1} \otimes \Box p_1 \otimes \Box p_3 \otimes \bigcirc\Box p_2 \to \mathbf{1} \otimes \Box p_1 \otimes \Box p_3 \otimes \bigcirc\Box p_2}{\tau_1^*, \tau_2^*; \mathbf{1} \otimes \Box p_1 \otimes \Box p_3 \otimes \Box p_3 \to \mathbf{1} \otimes \Box p_1 \otimes \Box p_3 \otimes \bigcirc\Box p_2}\ (fire)}{\tau_1^*, \tau_2^*; \mathbf{1} \otimes \Box p_1 \otimes \bigcirc\Box p_3 \otimes \bigcirc\Box p_3 \to \bigcirc(\mathbf{1} \otimes \Box p_1 \otimes \Box p_3 \otimes \bigcirc\Box p_2)}\ (next)}{\tau_1^*, \tau_2^*; \mathbf{1} \otimes \Box p_1 \otimes \bigcirc^2\Box p_3 \otimes \bigcirc^2\Box p_3 \to \bigcirc^2(\mathbf{1} \otimes \Box p_1 \otimes \Box p_3 \otimes \bigcirc\Box p_2)}\ (next)}{\tau_1^*, \tau_2^*; \mathbf{1} \otimes \Box p_1 \otimes \Box p_2 \otimes \Box p_2 \to \bigcirc^2(\mathbf{1} \otimes \Box p_1 \otimes \Box p_3 \otimes \bigcirc\Box p_2)}\ (fire)$$

Let us consider another Horn sequent with respect to Fig.1,

$$\tau_1^*, \tau_2^*; \mathbf{1} \otimes \bigcirc^2\Box p_3 \to \bigcirc^3(\mathbf{1} \otimes \Box p_2). \tag{2}$$

This is provable in **HTPN**;

$$\cfrac{\cfrac{\cfrac{\cfrac{\tau_1^*, \tau_2^*; \mathbf{1} \otimes \Box p_2 \to \mathbf{1} \otimes \Box p_2}{\tau_1^*, \tau_2^*; \mathbf{1} \otimes \bigcirc\Box p_2 \to \bigcirc(\mathbf{1} \otimes \Box p_2)}\ (next)}{\tau_1^*, \tau_2^*; \mathbf{1} \otimes \Box p_3 \to \bigcirc(\mathbf{1} \otimes \Box p_2)}\ (fire)}{\tau_1^*, \tau_2^*; \mathbf{1} \otimes \bigcirc\Box p_3 \to \bigcirc^2(\mathbf{1} \otimes \Box p_2)}\ (next)}{\tau_1^*, \tau_2^*; \mathbf{1} \otimes \bigcirc^2\Box p_3 \to \bigcirc^3(\mathbf{1} \otimes \Box p_2)}\ (next)$$

Suppose $S_1 = \langle \emptyset, \emptyset, \{p_3\}, \emptyset, \ldots \rangle$ and $S_2 = \langle \{p_2\}, \emptyset, \ldots \rangle$. By Lemma 8, $S_1\ [\sigma\overset{3}{\rangle}\ S_2$ for some σ. Furthermore, we can construct $\sigma = \delta\delta\tau_2\delta$ from the proof figure.

Lemma 8 can be extended to the following lemma;

Lemma 9 (Main lemma). *Let (Pl, Tr, Ar, θ) be TPN and S, S' states of it. Suppose Δ^* is a multiset structured from $\tau^* \in Tr^*$. If the Horn sequent*

$$Tr^*; \Delta^*, \mathbf{1} \otimes S^* \to \bigcirc^t(\mathbf{1} \otimes S'^*)$$

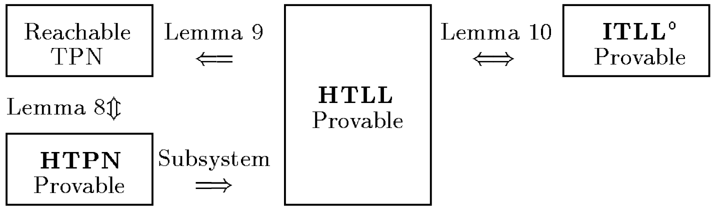

Fig.2: Illustration of the proof of Theorem 11

is provable in **HTLL** *then there is* σ *such that* $S \ [\sigma\overset{t}{\rangle}\ S'$ *and any* $\tau \in \Delta$ *has been really used in* σ *(i.e. For any* $\tau \in \Delta$, $\tau \in \sigma$*).* ■

Proof. (sketch) The claim is shown by induction on the height of the proof of the Horn sequent. See [7]. (Q.E.D.)

Let **ITLL**$^\circ$ be a subsystem of **ITLL** by replacing $(\rightarrow \otimes)$ with $(\rightarrow \otimes)^\circ$ and provided that all atomics are of the form $\Box p$, where

$$\frac{\Gamma, A \rightarrow A \quad \Delta, B \rightarrow B}{\Gamma, \Delta, A, B \rightarrow A \otimes B}\ (\rightarrow \otimes)^\circ$$

We can associate **HTLL** with **ITLL**$^\circ$ by the following lemma.

Lemma 10. *Let* $\Gamma; \Delta, M \rightarrow \bigcirc^t Z$ *be a Horn sequent. Then* $\Gamma; \Delta, M \rightarrow \bigcirc^t Z$ *is provable in* **HTLL** *if and only if* $!\Gamma, \Delta, M \rightarrow \bigcirc^t Z$ *is provable in* **ITLL**$^\circ$*.* ■

Proof. It is not difficult to show that if $\Gamma; \Delta, M \rightarrow \bigcirc^t Z$ is provable in **HTLL** then $!\Gamma, \Delta, M \rightarrow \bigcirc^t Z$ is provable in **ITLL**$^\circ$.

We sketch the proof of converse. Suppose $!\Gamma, \Delta, M \rightarrow \bigcirc^t Z$ is provable in **ITLL**$^\circ$ and $M = \alpha_1 \otimes \ldots \otimes \alpha_n$, where each α_i indicates a token formula. Then there exists some cut free proof of $!\Gamma, \Delta, \alpha_1, \ldots, \alpha_n \rightarrow \bigcirc^t Z$. One can prove the claim by the induction on the height of the proof figure. (Q.E.D.)

Now, we obtain the main theorem.

Theorem 11 (Completeness theorem). *Let* (Pl, Tr, Ar, θ) *be a timed Petri net and* S, S' *states of it. Then* S' *is reachable from* S *after* t *passage of time if and only if the sequent*

$$!Tr^*, \mathbf{1} \otimes S^* \rightarrow \bigcirc^t(\mathbf{1} \otimes S'^*)$$

is provable in **ITLL**$^\circ$*.* ■

Proof. (See Fig.2)

(Soundness) Suppose $!Tr^*, \mathbf{1} \otimes S^* \rightarrow \bigcirc^t(\mathbf{1} \otimes S'^*)$ is provable in **ITLL**$^\circ$. By Lemma 10, $Tr^*; \mathbf{1} \otimes S^* \rightarrow \bigcirc^t(\mathbf{1} \otimes S'^*)$ is provable in **HTLL**. Then $S \ [\sigma\overset{t}{\rangle}\ S'$ for some derivation sequence σ by Lemma 9.

(Completeness) Suppose $S \overset{t}{[\sigma\rangle} S'$ for some σ. $Tr^*; \mathbf{1} \otimes S^* \to \bigcirc^t(\mathbf{1} \otimes S'^*)$ is provable in **HTPN** by Lemma 8. Therefore, it is provable in **HTLL**. By Lemma 10, $!Tr^*, \mathbf{1} \otimes S^* \to \bigcirc^t(\mathbf{1} \otimes S'^*)$ is provable in **ITLL**$^\circ$. (Q.E.D.)

Unlike theorem 1, we have to restrict the tensor rule for the equivalence between the reachability of TPN and the provability of the corresponding sequent of temporal linear logic. This concludes that the following does not satisfy generally; if there exists some σ_1 such that $S_0[\sigma_1\rangle S$ and σ_2 such that $S'_0[\sigma_2\rangle S'$, then there exists σ such that $S_0 \uplus S'_0[\sigma\rangle S \uplus S'$. One can deduce $\Gamma_1, \Gamma_2, \bigcirc A, \bigcirc B \to \bigcirc(A \otimes B)$ from $\Gamma_1, \bigcirc A \to \bigcirc A$ and $\Gamma_2, \bigcirc B \to \bigcirc B$ in **ITLL**$^\circ$. This concludes that if $!Tr^*, \mathbf{1} \otimes S^*_0 \to \bigcirc^t(\mathbf{1} \otimes S^*)$ and $!Tr^*, \mathbf{1} \otimes S'^*_0 \to \bigcirc^t(\mathbf{1} \otimes S'^*)$ are provable in **ITLL**$^\circ$ then $!Tr^*, \mathbf{1} \otimes S^*_0 \otimes S'^*_0 \to \bigcirc^t(\mathbf{1} \otimes S^* \otimes S'^*)$ is also provable in **ITLL**$^\circ$. We can say that if we match between the passages of time then we can combine two derivation sequences.

5 The Decidability of the Reachability Problem for Timed Petri Nets

By the previous section, the strict reachability for timed Petri nets is equivalent to the provability of the corresponding Horn sequent. In this section, we show the decidability of the strict reachability problem for timed Petri nets by rewriting a Horn sequent (Corollary 13).

Let $\mathcal{S}$ be the following Horn sequent of **HTPN**;

$$\Gamma; \mathbf{1} \otimes M \to \bigcirc^t(\mathbf{1} \otimes Z),$$

where Γ is a set of formulas of the form $X \multimap Y$. We rewrite $\mathcal{S}$ to obtain the rewritten Horn sequent $\tilde{\mathcal{S}}$

$$\tilde{\Gamma}; clock^{(0)} \otimes \tilde{M} \to clock^{(t)} \otimes \tilde{Z},$$

which does not include temporal modalities. The rewriting steps are as follows;

Rewriting steps for a Horn sequent

- Rewriting for M and Z.
  1. Each $\mathbf{1}$ on both sides is removed. We put an atomic $clock^{(0)}$ in front of M and an atomic $clock^{(t)}$ instead of $\bigcirc^t$.
  2. Each token fomula of the form $\bigcirc^k\Box p$ in M and Z is rewritten into an atomic $p^{(k)}$ and $p^{(t+k)}$, respectively.
- Rewriting for Γ. The rewriting corresponds to the translation from TPN structure into PN structure.
  1. Each $X \multimap Y$ is rewritten into a series of linear implications of the forms
  $$clock^{(i)} \otimes X \multimap clock^{(i)} \otimes Y,$$
  where $0 \leq i \leq t$. Each token fomula of the form $\bigcirc^k\Box p$ in X and Y is rewritten into an atomic $p^{(i+k)}$.

2. We add a series of auxiliary linear implications to Γ.
   (a) We add a series of auxiliary linear implications of the forms

$$clock^{(j)} \multimap tmp^{(j)}, tmp^{(j)} \multimap clock^{(j+1)},$$

   where $0 \leq j \leq t-1$.
   (b) For each p in X and Y, we add a series of auxiliary linear implications of the form

$$tmp^{(j)} \otimes p^{(j)} \multimap tmp^{(j)} \otimes p^{(j+1)}$$

   where $0 \leq j \leq t-1$.

For example, we consider a Horn sequent (1) in section 4 as $\mathcal{S}$. We can rewrite it into the following Horn sequent as $\tilde{\mathcal{S}}$;

$$\tilde{Tr}; clock^{(0)} \otimes p_1^{(0)} \otimes p_2^{(0)} \otimes p_2^{(0)} \to clock^{(2)} \otimes p_1^{(2)} \otimes p_3^{(2)} \otimes p_2^{(3)} \tag{3}$$

where $\tilde{Tr}$ is the following sequence of linear implications;

$$\begin{array}{l} clock^{(i)} \otimes p_1^{(i)} \otimes p_2^{(i)} \otimes p_2^{(i)} \multimap clock^{(i)} \otimes p_1^{(i)} \otimes p_3^{(i+2)} \otimes p_3^{(i+2)}, \\ clock^{(i)} \otimes p_3^{(i)} \multimap clock^{(i)} \otimes p_2^{(i+1)}, \\ clock^{(j)} \multimap tmp^{(j)}, tmp^{(j)} \multimap clock^{(j+1)}, \\ tmp^{(j)} \otimes p_1^{(j)} \multimap tmp^{(j)} \otimes p_1^{(j+1)}, \\ tmp^{(j)} \otimes p_2^{(j)} \multimap tmp^{(j)} \otimes p_2^{(j+1)}, \\ tmp^{(j)} \otimes p_3^{(j)} \multimap tmp^{(j)} \otimes p_3^{(j+1)} \qquad (0 \leq i \leq 3,\ 0 \leq j \leq 2). \end{array}$$

We can obtain the following lemma by replacing (*next*) rules in the proof figure of $\mathcal{S}$ into (*fire*) rules with respect to auxiliary linear implications in $\tilde{\Gamma}$;

Lemma 12. *For a given Horn sequent $\mathcal{S}$ of* **HTPN**, *suppose $\tilde{\mathcal{S}}$ is the rewritten Horn sequent. Then we can say that $\mathcal{S}$ is provable in* **HTPN** *if and only if $\tilde{\mathcal{S}}$ is provable in* **HTPN** *without* (*next*) *rule.* ∎

For the proof figure of (1) on page 6, the corresponding proof figure of (3) without (*next*) rule is as follows;

$$\dfrac{\dfrac{\dfrac{\dfrac{\tilde{Tr}; clock^{(2)} \otimes p_1^{(2)} \otimes p_3^{(2)} \otimes p_2^{(3)} \to clock^{(2)} \otimes p_1^{(2)} \otimes p_3^{(2)} \otimes p_2^{(3)}}{\tilde{Tr}; clock^{(2)} \otimes p_1^{(2)} \otimes p_3^{(2)} \otimes p_3^{(2)} \to clock^{(2)} \otimes p_1^{(2)} \otimes p_3^{(2)} \otimes p_2^{(3)}}\ 4}{\tilde{Tr}; clock^{(1)} \otimes p_1^{(1)} \otimes p_3^{(2)} \otimes p_3^{(2)} \to clock^{(2)} \otimes p_1^{(2)} \otimes p_3^{(2)} \otimes p_2^{(3)}}\ 3((\text{fire})3 \text{ times})}{\tilde{Tr}; clock^{(0)} \otimes p_1^{(0)} \otimes p_3^{(2)} \otimes p_3^{(2)} \to clock^{(2)} \otimes p_1^{(2)} \otimes p_3^{(2)} \otimes p_2^{(3)}}\ 2((\text{fire})3 \text{ times})}{\tilde{Tr}; clock^{(0)} \otimes p_1^{(0)} \otimes p_2^{(0)} \otimes p_2^{(0)} \to clock^{(2)} \otimes p_1^{(2)} \otimes p_3^{(2)} \otimes p_2^{(3)}}\ 1$$

Double lines mean that several inference rules are applied.

Each number 1 – 4 in the proof figure means one or several (*fire*) rules which correspond to the following linear implications, respectively;

1. $clock^{(0)} \otimes p_1^{(0)} \otimes p_2^{(0)} \otimes p_2^{(0)} \multimap clock^{(0)} \otimes p_1^{(0)} \otimes p_3^{(2)} \otimes p_3^{(2)}$.
2. $clock^{(0)} \multimap tmp^{(0)}$, $tmp^{(0)} \otimes p_1^{(0)} \multimap tmp^{(0)} \otimes p_1^{(1)}$, $tmp^{(0)} \multimap clock^{(1)}$.
3. $clock^{(1)} \multimap tmp^{(1)}$, $tmp^{(1)} \otimes p_1^{(1)} \multimap tmp^{(1)} \otimes p_1^{(2)}$, $tmp^{(1)} \multimap clock^{(2)}$.
4. $clock^{(2)} \otimes p_3^{(2)} \multimap clock^{(2)} \otimes p_2^{(3)}$.

Lemma 12 concludes that the strict reachability problem for timed Petri nets can be translated into the reachability problem for Petri nets. Since the reachability problem for Petri nets is decidable [9], we can obtain the following corollary which claims that the strict reachability problem for timed Petri nets is decidable.

Corollary 13 (Decidability of the strict reachability problem). *Let S and S' be states of a timed Petri net, $t \in \mathbf{N}$. We can decide if $S' \in [S\rangle_t$.*
∎

This result is similar to [12, 13].

6 Conclusions and Future Work

We have obtained the result that the reachability problem for TPN is equivalent to the provability problem for the corresponding sequent of the subsystem of temporal linear logic. Unlike in Theorem 1, we restrict the $\otimes$-rule of temporal linear logic. It is caused by the difference between markings and states. Although firing sequences in PN are able to be combined, we have to match between the time informations of states to combine derivation sequences in TPN.

It is not difficult to show that the sequent $!Tr^*, \mathbf{1} \otimes S^* \to \bigcirc^t(\mathbf{1} \otimes \top \otimes S'^*)$ is provable in $\mathbf{ITLL}^\circ$ if and only if "The state which includes S' is reachable from S after t passage of time".

As compared with [4], we will be able to obtain classical temporal linear logic. Since we have a fragment $\Diamond$, which means "sometimes"[4], in full fragments of classical temporal linear logic, it will provide an expressive power such as "The state is sometimes reachable from the initial state".

It is known that the reachability problem for PN is decidable [9]. Since TPN can be translated into PN, the reachability problem for TPN is also decidable [12, 13]. We can obtain the similar result by logical method.

To use & allows us to express a token which can be used for a fixed period of time. For example, $p \& \bigcirc p \& \bigcirc^2 p$ means that only during two units of passage of time, we can use the token p. The expressive powers of temporal linear logic will be helpful to our final target to analyze the dynamic behavior of timed Petri nets.

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References

1. C. Brown. Petri nets as quantales. Technical Report ECS-LFCS-89-96, Department of Computer Science, University of Edinburgh, November 1989.
2. C.Ramchandani. Analysis of Asynchronous Concurrent Systems by Timed Petri Nets, 1974. Project MAC, TR120, MIT.
3. Jean-Yves Girard. Linear logic. *Theoretical Computer Science*, 50:1–102, 1987.
4. Max Kanovich and Takayasu Ito. Temporal linear logic specifications for concurrent processes (extended abstract). In *Twelfth Annual IEEE Symposium on Logic in Computer Science*, pages 48–57, Warsaw, Poland, 29 June–2 July 1997. IEEE Computer Society Press.
5. Max I. Kanovich. The Horn fragment of linear logic is NP-complete. ITLI Prepublication Series X-91-14, University of Amsterdam, 1991.
6. Max I. Kanovich. Horn programming in linear logic is NP-complete. In *Seventh Annual Symposium on Logic in Computer Science*, pages 200–210, Santa Cruz, California, June 1992. IEEE Computer Society Press.
7. Max I. Kanovich. Linear logic as a logic of computations. *Annals of Pure and Applied Logic*, 67(1-3):183–212, 1994. Also in *Logic at Tver '92*, Sokal, Russia, July 1992.
8. J. Lilius. High-level nets and linear logic. In K. Jensen, editor, *Proceedings of the International Conference on Applications and Theory of Petri Nets*, pages 310–327, Sheffield, United Kingdom, June 1992. Springer-Verlag LNCS 616.
9. E. W. Mayr. An algorithm for the general Petri net reachability problem. In *Proc. of the 13th Annual ACM Symp. on Theory of Computing*, pages 238–246, 1981. Second Edition: SIAM J. Comput. Vol. 13, No. 3, Pages: 441-460, August 1984.
10. J. Meseguer and U. Montanari. Petri nets are monoids, 1990. INFORMATION AND COMPUTATION, 88:105-155.
11. Amir Pnueli. The temporal logic of programs. In *18-th Annual Symposium on the Foundations of Computer Science*, pages 46–57, 1977.
12. V. Valero Ruíz, D. de Frutos Escrig, and F. Cuartero Gómez. Simulation of timed Petri nets by ordinary Petri nets and applications to decidability of the timed reachability problem and other related problems. In *Proceedings of the Fourth International Workshop on Petri Nets and Performance Models (PNPM91)*, pages 154–163, December 2-5 1991.
13. V. Valero Ruíz, D. de Frutos Escrig, and F. Cuartero Gómez. Decidability of the strict reachability problem for TPN's with rational and real durations. In *5th International Workshop on Petri Nets and Performance Models*, pages 56–65,19.–22, Toulouse (F), October 1993.
14. Makoto Tanabe. Timed Petri nets and temporal linear logic. *Lecture Notes in Computer Science*, 1248:156–174, June 1997. 18th International Conference on Application and Theory of Petri Nets, Toulouse, France, June 1997.
15. U.Engberg and G. Winskel. Linear logic on Petri nets. *LNCS 803*, pages 176–229, Jun 1993. In A Decade of Concurrency, Reflections and Perspectives, REX School/Symposium.

A Syntax for Intuitionistic Temporal Linear Logic

Roman capitals $A, B, \ldots$ stand for formulas. The connectives of propositional temporal linear logic are:

- the *multiplicatives* $A \otimes B, A \multimap B, \mathbf{1}$;
- the *additives* $A \& B, A \oplus B, \top, \mathbf{0}$;
- the *exponentials* $!A$;
- the *temporal modalities* $\Box A, \bigcirc A$.

Greek capitals $\Gamma, \Pi, \ldots$ stand for sequents, which are multisets of formulas, so that exchange is implicit. **ITLL** is defined as follows;

Identity and Cut rule:

$$\frac{}{D \to D}\ (I) \qquad \frac{\Gamma \to D \quad D, \Pi \to C}{\Gamma, \Pi \to C}\ (cut)$$

Propositional Rules :

$$\frac{A, B, \Gamma \to C}{A \otimes B, \Gamma \to C}\ (\otimes \to) \qquad \frac{\Gamma \to A \quad \Pi \to B}{\Gamma, \Pi \to A \otimes B}\ (\to \otimes)$$

$$\frac{A, \Gamma \to C}{A \& B, \Gamma \to C}\ (\& \to)1 \quad \frac{B, \Gamma \to C}{A \& B, \Gamma \to C}\ (\& \to)2 \quad \frac{\Gamma \to A \quad \Gamma \to B}{\Gamma \to A \& B}\ (\to \&)$$

$$\frac{A, \Gamma \to C \quad B, \Gamma \to C}{A \oplus B, \Gamma \to C}\ (\oplus \to) \quad \frac{\Gamma \to A}{\Gamma \to A \oplus B}\ (\to \oplus)1 \quad \frac{\Gamma \to B}{\Gamma \to A \oplus B}\ (\to \oplus)2$$

$$\frac{\Gamma \to A \quad B, \Pi \to C}{A \multimap B, \Gamma, \Pi \to C}\ (\multimap \to) \qquad \frac{A, \Gamma \to B}{\Gamma \to A \multimap B}\ (\to \multimap)$$

Constants :

$$\frac{\Gamma \to C}{\mathbf{1}, \Gamma \to C}\ (\mathbf{1} \to) \quad \frac{}{\to \mathbf{1}}\ (\to \mathbf{1}) \quad \frac{}{\Gamma, \mathbf{0} \to C}\ (\mathbf{0} \to) \quad \frac{}{\Gamma \to \top}\ (\to \top)$$

Exponential Rules :

$$\frac{A, \Gamma \to C}{!A, \Gamma \to C}\ (! \to) \quad \frac{!\Gamma \to A}{!\Gamma \to !A}\ (\to !) \quad \frac{\Gamma \to C}{!A, \Gamma \to C}\ (!w) \quad \frac{!A, !A, \Gamma \to C}{!A, \Gamma \to C}\ (!c)$$

Modality Rules :

$$\frac{A, \Gamma \to C}{\Box A, \Gamma \to C}\ (\Box \to) \quad \frac{!\Gamma, \Box\Pi \to A}{!\Gamma, \Box\Pi \to \Box A}\ (\to \Box) \quad \frac{!\Gamma, \Box\Pi, \Xi \to A}{!\Gamma, \Box\Pi, \bigcirc\Xi \to \bigcirc A}\ (\bigcirc)$$

Table 2. propositional intuitionistic linear logic **ITLL**

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